ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ СИБИРСКОГО ОТДЕЛЕНИЯ АН СССР

препринт

DOUBLE BREMSSTRAHLUNG IN COLLIDING BEAM EXPERIMENTS

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НОВОСИБИРСК 1965

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Preprint

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Of considerable interest in the experiments with colliding beams is the process of double bremsstrahlung, i.e. the process in which the collosion between electrons or between electron and positron is accompanied by the emission of two photons. This process can be used as a monitor for the registration of beam collisions. The simplest scheme seems to be such in which quanta emitting in opposite directions are registered. With the availability of such a scheme the process of the double bremsstrahlung competes with the process of two-quanta annihilation (the case of electron-positron collisions). In case of high-energy electrons and within a sufficiently wide range of frequencies of photons under registration, the cross-section of this process can exceed that of two-quanta annihilation. This is connected with the fact that the cross-section of the double bremsstrahlung in contrast to that of two-quanta annihilation does not decrease with the increase of the colliding particle energy.

The use of double bremsstrahlung as a monitor requires the knowledge of theoretical formulas for the cross-section of this process with a sufficient degree of accuracy. Taking into account that photon emission occurs mainly into the angle of $1/\gamma$ order and that it is less than the angular dimension counters at large energies, it becomes clear that the cross-section integrated over the angles of photon emission is of interest. Since electrons are not registered in their final state, it is also necessary to carry out the integration over their final states. The quantity thus obtained will be a differential cross-section over both quanta frequencies of the double bremsstrahlung $d\sigma_{\omega_1\omega_2}$ where ω_1 is the frequency of a photon emitted in the direction 1, and ω_2 is the frequency of the photon emitted in the direction 2, opposite to the direction 1.

The calculation of this cross-section for the case of classical quanta bremsstrahlung / $\omega_{1,2} \ll \varepsilon$ / is performed in ref. [1] , and that for the case when only one of the emitted photons is soft while the second has an arbitrary energy obtained in ref. [2]. The paper reports the results of the calculation of the cross-section of the double bremsstrahlung of photons with arbitrary energy. It is assumed that the electron energy is high so that the expansion in the inverse powers γ can be performed.

2. The above formulation of the question naturally determines the choice of diagrams. In fact, since the small scattering angles are of importance, and each of electrons emits into a narrow cone with the angle of the order of $1/\gamma$, the main contribution to the cross-section is given by the diagrams corresponding to the emission of quanta by different particles. Since the interference is non-essential one is to consider four diagrams presented in the figure. The results obtained will be equally valid for electron-electron and electron-positron scattering.

It turns to be reasonable to separate the integration over the final states of each electron and over the angles of the photon emitted by it by introducing the additional δ-function. Thereby the cross-section of the double bremsstrahlung will be x)

$$d\sigma_{\omega_1\omega_2} = \frac{4\alpha^4}{(2\pi)^4\sqrt{(p_1p_2)^2-1}} \frac{d\omega_1}{\omega_1}\frac{d\omega_2}{\omega_2}\int \frac{d^4\Delta}{\Delta^4} K_{1\mu\nu} K_2^{\mu\nu} \quad (1)$$

x) Here and in what follows one uses the metric $(ab) = (\vec{a}\vec{b}) - a_0 b_0$ and the system of unities $\hbar = c = m = 1$.

Quantity $K_{1\mu\nu}$ is proportional to the scattering cross-section of the arbitrary polarized photon on electron integrated over all the final states, except for frequency, with the square of mass of the initial photon being $-\Delta^2$. Tensors $K_{\mu\nu}$ depend on the reference system in which the quanta energy ω is fixed. In order to write down them in a covariant form it is convenient to introduce 4-vector n_μ defined so that in the reference system being of interest to us (c.m.s.) its components are equal to $n_\mu = (0,0,0,1)$. Then, the tensor $K_{\mu\nu}$ can be expressed in terms of vectors p_μ , Δ_μ , n_μ , with the coefficients by their quadratic combinations defined by the following four covariant integrals

$$J_1^{(i)} = g^{\mu\nu} K_{i\mu\nu}\,, \quad J_2 = p_i^\mu p_i^\nu K_{i\mu\nu}\,,$$
$$J_3^{(i)} = n^\mu n^\nu K_{i\mu\nu}\,, \quad J_4^{(i)} = (n^\mu p_i^\nu + n^\nu p_i^\mu) K_{i\mu\nu} \quad (2)$$
$$/\, i = 1,2 \,/$$

3. In the subsequent calculations one makes use of the smallness of the parameter ε^{-2} . This approximation is consistent as the dropped diagrams give the contribution of the above order. In this approximation the only essential term in the product of tensors of the formula (1) proves to be the product $J_3^{(1)} \cdot J_3^{(2)}$. This is due to the fact that in the reduction of the matrix elements with the vector n_μ there appear terms of the form (np) and (nk) whose order is ε while the products of the form (pp) and (kp) are of the order of the unity. Hence, the most essential are those summands which contain the maximal number of vectors n_μ , i.e. "the least covariant terms".

The quantity J_3 has a transparent physical meaning. It is the 00-component of the tensor $K_{\mu\nu}$, namely the component entering the cross-section of the electron bremsstrahlung on the Coulomb center.

The integration over Δ_0 and Δ_z (Δ_z is the component of the vector Δ parallel to the direction of of the initial electron momentum)is carried out simply.On choosing the terms of the highest order over a small parameter ε^{-2} the result of integration is written down as

$$d\sigma_{\omega_1\omega_2} = \frac{16\alpha^4}{\pi}\frac{d\omega_1}{\omega_1}\frac{d\omega_2}{\omega_2}\int\frac{d\Delta^2}{\Delta^4}\left\{\left(1-\frac{\omega_1}{\varepsilon}\right)\Phi\left(\frac{\Delta^2}{4}\right)+\frac{\omega_1^2}{\varepsilon^2}\frac{\Delta}{\sqrt{\Delta^2+4}}\ln\left(\frac{\Delta}{2}+\sqrt{1+\frac{\Delta^2}{4}}\right)\right\}\times$$
$$\times\left\{\left(1-\frac{\omega_2}{\varepsilon}\right)\Phi\left(\frac{\Delta^2}{4}\right)+\frac{\omega_2^2}{\varepsilon^2}\frac{\Delta}{\sqrt{\Delta^2+4}}\ln\left(\frac{\Delta}{2}+\sqrt{1+\frac{\Delta^2}{4}}\right)\right\} \qquad (3)$$

$$\Phi(x^2) = \frac{1+2x^2}{x\sqrt{1+x^2}}\ln\left(x+\sqrt{1+x^2}\right)-1$$

At $\omega \to 0$ the braced expressions are equal to $\Phi\left(\frac{\Delta^2}{4}\right)$, i.e. the quantity proportional to the probability of the emission of the classical photon integrated over the angles of photon emisison in the Δ momentum transfer to the electron. Therefore these expressions may be regarded as a generalization of such a probability to the case of arbitrary energy photons. For small Δ such a generalized probability is proportional to Δ^2 so that in the integral of the formula (3) the small are non-essential, and the lower limit of integration may be put equal to zero. This implies the inapplicability of Weizsecker-Williams method for this problem.

The upper integration limit over Δ^2 is proportional to ε^2 and in view of convergence of the integral may be put equal to infinity. On integration one obtains the following resulting formula for the cross-section of the double bremsstrahlung

$$d\sigma_{\omega_1,\omega_2} = \frac{8r_0^2\alpha^2}{\pi}\left\{\left(1-\frac{\omega_1}{\varepsilon}\right)\left(1-\frac{\omega_2}{\varepsilon}\right)\left[\frac{5}{4}+\frac{7}{8}\zeta(3)\right]+\right.$$
$$\left.+\left[\left(1-\frac{\omega_1}{\varepsilon}\right)\frac{\omega_2^2}{\varepsilon^2}+\left(1-\frac{\omega_2}{\varepsilon}\right)\frac{\omega_1^2}{\varepsilon^2}\right]\left[\frac{1}{2}+\frac{7}{8}\zeta(3)\right]+\frac{\omega_1^2\omega_2^2}{\varepsilon^4}\frac{7}{8}\zeta(3)\right\}\frac{d\omega_1}{\omega_1}\frac{d\omega_2}{\omega_2}. \qquad (4)$$

$$\frac{7}{8}\zeta(3) = 1{,}052.$$

At $\omega_{1,2} \ll \varepsilon$ this expression is reduced to the formula (22) of the ref. [1] in which the numerical coefficient is not correct and is to be 4 times decreased.

Formula (4) is invalid for the most hard part of the spectrum when $\varepsilon - \omega_{1,2}$ is of the order of unity. However, in view of the narrowness of the interval this region gives no final contribution to the integral cross-section.

The estimation of the dropped terms in the expression (4) for the quanta of the arbitrary energy turns to be sufficiently complex. However, this estimation is easily carried out for the soft quanta $\omega_{1,2} \ll \varepsilon$. Then the correction to the cross-section is of the order

$$\delta\sigma \sim r_0^2\alpha^2\,\frac{\ln^3\varepsilon}{\varepsilon^2}\,\frac{d\omega_1}{\omega_1}\frac{d\omega_2}{\omega_2} \qquad (5)$$

When electron energy is 50 Mev it is 2-3% and decreases rapidly with the increase of energy.

R E F E R E N C E S.

1. V. Bayer, V. Galitsky, Phys.Lett., 13, 355 /1964/.
2. V.N. Bayer and V.M. Galitsky, JETP, 49, 2 /1965/.

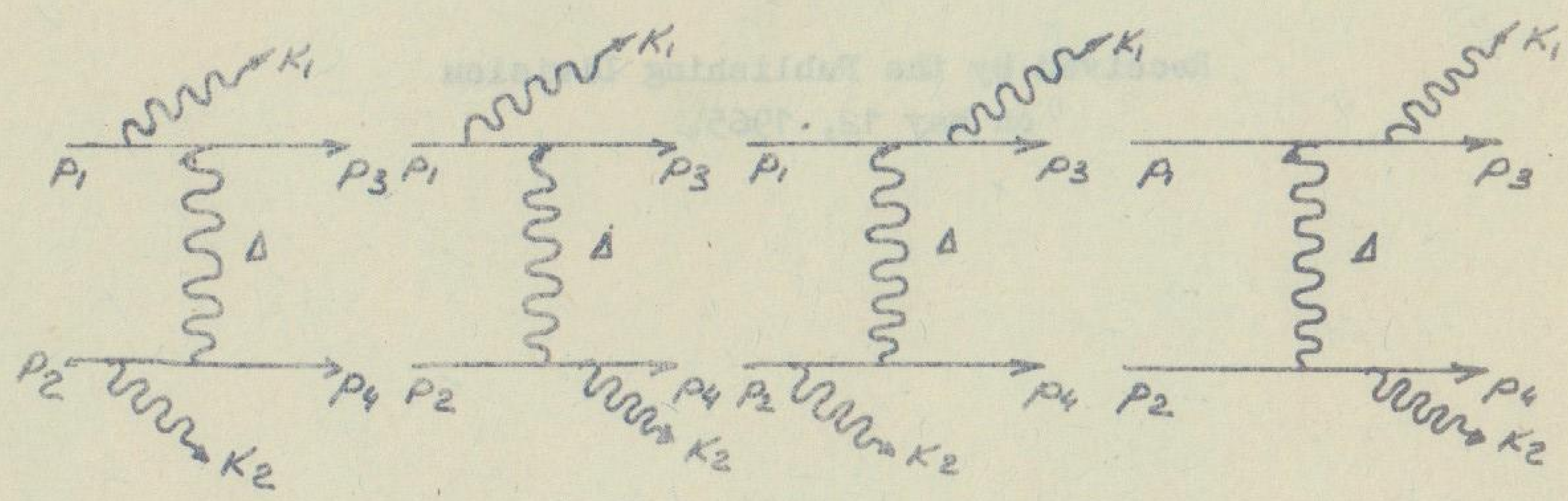

Fig.